Towards optimum Demodulation of Bandwidth-Limited and Low SNR Square-Wave Subcarrier Signals

Ying J. Feria   William J. Hurd

Jet Propulsion Laboratory
California Institute of Technology
4800 Oak Grove Drive
Pasadena, CA 91109-8099

Abstract

For the extremely weak signal received from the Galileo spacecraft, even a fraction of a dB in symbol signal-to-noise ratio (SNR) is significant to the mission. This work was motivated by the need for near-optimum demodulation of the Galileo signal. We present here the optimum phase detector jor tracking square-wave subcarriers which have been bandwidth limited to a finite number of harmonics. The phase detector is optimum in the sense that the loop signal-to-noise ratio (SNR) is maximized and hence the RMS phase tracking error is minimized. The optimum phase detector is easy to implement and achieves substantial improvement over the initial phase detector (0.1 to 0.3-dB improvement in symbol SNR). We also present the optimum weights to combine the signals demodulated from each of the harmonics. The optimum weighting provides an SNR improvement of 0.1 to 0.15 dB when the subcarrier loop SNR is low (15 all]) and the number of harmonics is high (8 to 16).

1 Introduction

The Galileo spacecraft has a very weak downlink signal due to failure of the high gain antenna to deploy. Therefore, there has been a major effort in the NASA Deep Space Network to insert new technology to improve data reception.

For the extremely weak signal received from the Galileo spacecraft, even a fraction of a dB in signal-to-noise ratio (SNR) is significant to the mission. To avoid the risk of loosing data when the tracking loops lose lock, wc usc a "full-spectrum" recorder to record the downconverted signal. '1'0 gain additional symbol

SNR, we record the same signal from various antennas, transmit them to a central location, align them, and combine them. For example, by adding a signal from a 34-meter antenna to that from a 70-meter antenna, we gain about 0.68 dB assuming perfect signal alignment.

Because the telemetry signal is on a square-lvavc subcarrier, the received signal is not band-limited. So, recording the full-spectrum signal is not practical, hence only the first four harmonics arc recorded by the Full Spectrum Recorder. The recorded signal then goes into the Buffered Telemetry Demodulator (BTD) that is specially designed to demodulate the first four harmonics of the square-wave subcarrier.

This work was motivated by the need for near-optimum demodulation in the BTD. Since the BTD is a software demodulator, it impractical to tailor the processing more closely to the Galileo signal conditions than would be practical in real-time h ardware systems, such as the Block V Receiver.

The subcarrier phase detector initially implemented in the BTD uses a windowing technique similar to that used in the Advanced Receiver 11 and the Block V Receiver [1] but modified for the four-harmoni c case. In the initial BTD phase detector, there is a parameter, W_{sc}, which is analogous to the fractional window width in a square-wave subcarrier phase detector. As shown in Fig. 1, this phase detector results in a degradation (loss in symbol SNR duc to harmonic truncation and phase tracking error), which dots not monotonically decrease as the number of harmonics increases [1]. In fact, when the tracking error is large, and when the harmonics arc combined using the usual 1/n weighting for the nth harmonic, it is sometimes better to usc only four harmonics than to use all harmonics. This suggests two things: first, it tells us that the phase detector may not be using the harmonics optimally; and second, it indicates that the demodulated harmonics may not be optimally combined.

The phase detector used in [3] is derived froı n a window used on a square-wave subcarrier loop. This phase detector may not be optimum for a finite-harmonic subcarrier.

As a previous work [2] indicates, the higher harmonics have larger phase noise jitters. Thcrcforc, the effective signal amplitude on the n-th harmonic is no longer $1/n$ but some number smaller than that. The optimum weights to combine the demodulated harmonics should account for the SNR losses due to the loop.

2 Optimum Phase Detector

Here we derive a phase detector (PD) that is optimum in the sense that the

[1] Work done by 1). Rogstad and Y. Feria, Jet Propulsion Laboratory, Pasadena, CA., Oct. 1994

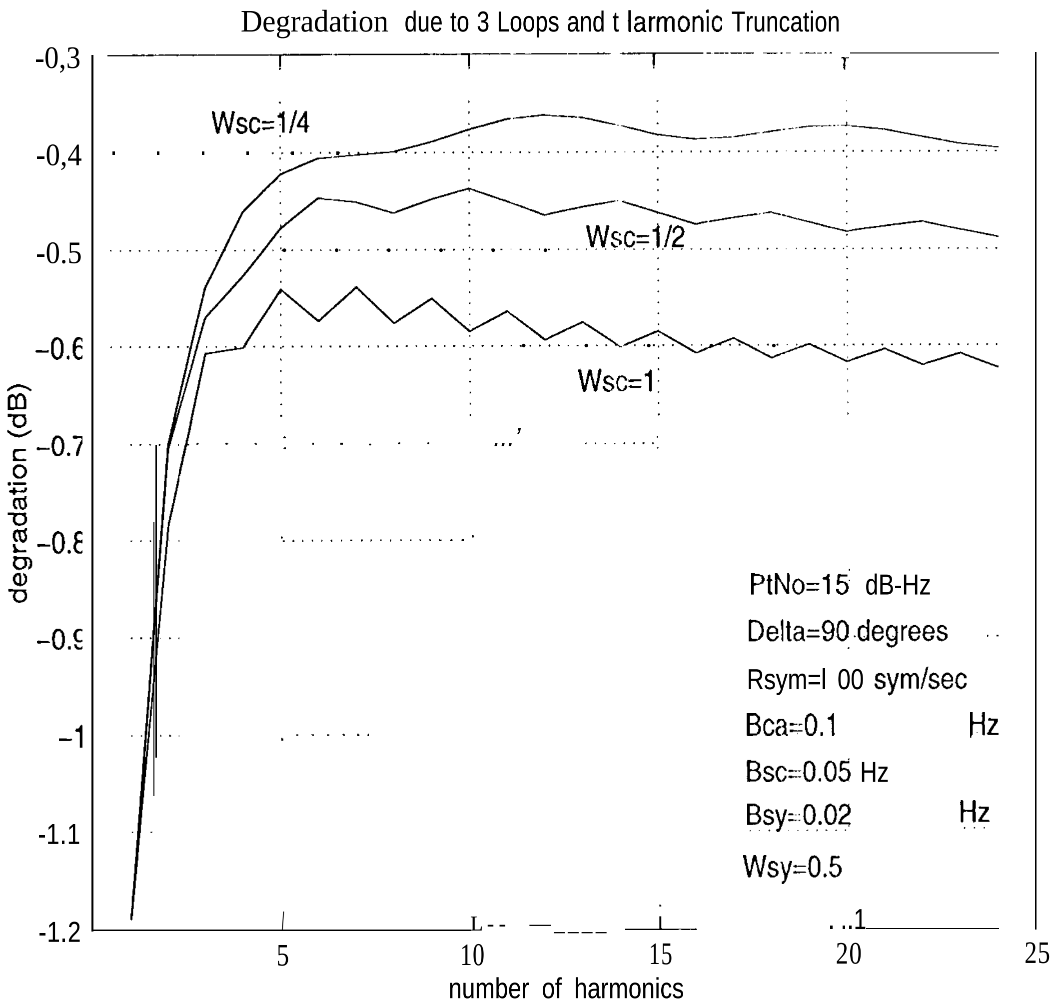

Figure 1: Degradations as function of number of harmonics using the initial BTD.

loop SNR is maximized. To show the derivation, let us first look at the phase detector initially used in BTD. The initial phase detector is the product of the combined in-phase signals $\sqrt{P_d}d_k \cos\phi_c \frac{8}{\pi^2}\sum_{n=0}^{L-1}\frac{1}{(2n+1)^2}\cos[(2n+1)\phi_{sc}]$ and the combined quadrature signals $\sqrt{P_d}d_k \cos\phi_c \frac{8}{\pi^2}\sum_{n=0}^{L-1} w_n \frac{1}{2n+1}\sin[(2n+1)\phi_{sc}]$, where the w_n's are the weights used to combine the quadrature signals. In the initial phase detector in the BTD, these weights are

$$w_n = \frac{\sin[(2n+1)\frac{\pi}{2}W_{sc}]}{2n+1}$$

The loop SNR using the initial BTD is derived [2] as

$$\rho_{sc} = \frac{\alpha\beta^2}{\gamma B_{sc}}\frac{P_d}{N_o}\left(\alpha + \frac{1}{2E_s/N_o}\right)^{-1}$$

where

$$\alpha = \frac{8}{\pi^2}\sum_{n=0}^{L-1}\frac{1}{(2n+1)^2}$$

$$\beta = \frac{8}{\pi^2}\sum_{n=0}^{L-1} w_n$$

$$\gamma = \frac{8}{\pi^2}\sum_{n=0}^{L-1} w_n^2$$

where L is the total number of harmonics used in the phase detector, P_d/N_o is the data power-to-noise ratio, E_s/N_o is the symbol SNR, and B_{sc} is the subcarrier loop bandwidth.

Now in order to maximize the subcarrier loop SNR, ρ_{sc}, let $w_k, k = 0, \ldots, L-1$, be unknown and α be the same as before, and differentiate the loop SNR, ρ_{sc}, with respect to w_k and set the expression to zero. We then have,

$$\frac{\partial\rho_{sc}}{\partial w_k} = \frac{2\beta\gamma - 2\beta^2 w_k}{\gamma^2}\frac{1}{B_{sc}}\frac{P_d}{N_o}\frac{\alpha}{\alpha + 1/(2E_s/N_o)} = 0 \tag{1}$$

Since $P_d/N_o \neq 0$, $\alpha \neq 0$, and γ, B_{sc} are finite, the above is zero if and only if

$$\gamma - \beta w_k = 0$$

That is,

$$\sum_{n=0}^{L-1} w_n^2 - \sum_{n=0}^{L-1} w_n w_k = 0$$

[2] H. Tsou, Private Communication, Jet Propulsion Laboratory, Pasadena, CA, Oct. 1994.

or

$$\sum_{n=0}^{L-1} w_n(w_n - w_k) = 0, \text{ for all } k$$

which implies that

$$w_n = w_k, \text{ for all } n \text{ and } k$$

The conclusion is that the optimum weights to combine the quadrature signals in the phase detector are a constant for all (finite) harmonics. Note that, for, infinite number of harmonics, the parameters β and γ do not converge; therefore, the above weights cannot be used for square waves.

When the optimum weights are used in the phase detector, the loop SNR becomes

$$\rho_{sc} = \frac{L}{B_{sc}} \frac{P_d}{No} \frac{\alpha}{\alpha + 1/(2E_s/N_o)} \tag{2}$$

Using the optimum weights in the phase detector, and calling it the optimum phase detector, we can improve the loop SNR by 9.5 dB over the initial BTD with window size=1, and by 1.1 dB over the initial BTD with window size=1/4 (see Fig. 2). The same figure also shows that using the optimum phase detector, the loop SNR obtained by using only one harmonic is higher than that using the initial BTD with window size 1 or 1/2. Note that when we use only one harmonic in the optimum phase detector, we may still use all the available harmonics to demodulate the subcarrier.

Degradations due to a finite-harmonic subcarrier loop can be computed using the expressions given in [1]. Degradations as a function of the number of harmonics are shown in Fig. 4. Clearly, we can observe that using the optimum phase detector, we obtained a lower degradation with more harmonics. This agrees with our intuition.

With the increase of the loop SNR, that is, with the increase of number of harmonics, the linear region of the normalized S-curves shown in Fig. 3 shrinks. As the number of harmonics approaches infinity, the linear region of the S-curve vanishes. In other words, this optimum phase detector is valid only for a finite number of harmonics.

3 Optimum Combining Weights in Demodulation

The demodulated harmonics are normally combined with weight 1/n for the n-th harmonic. These weights are optimum if each of the harmonics of the subcarrier is demodulated with the same phase jitter. In our Case, however, we know that if

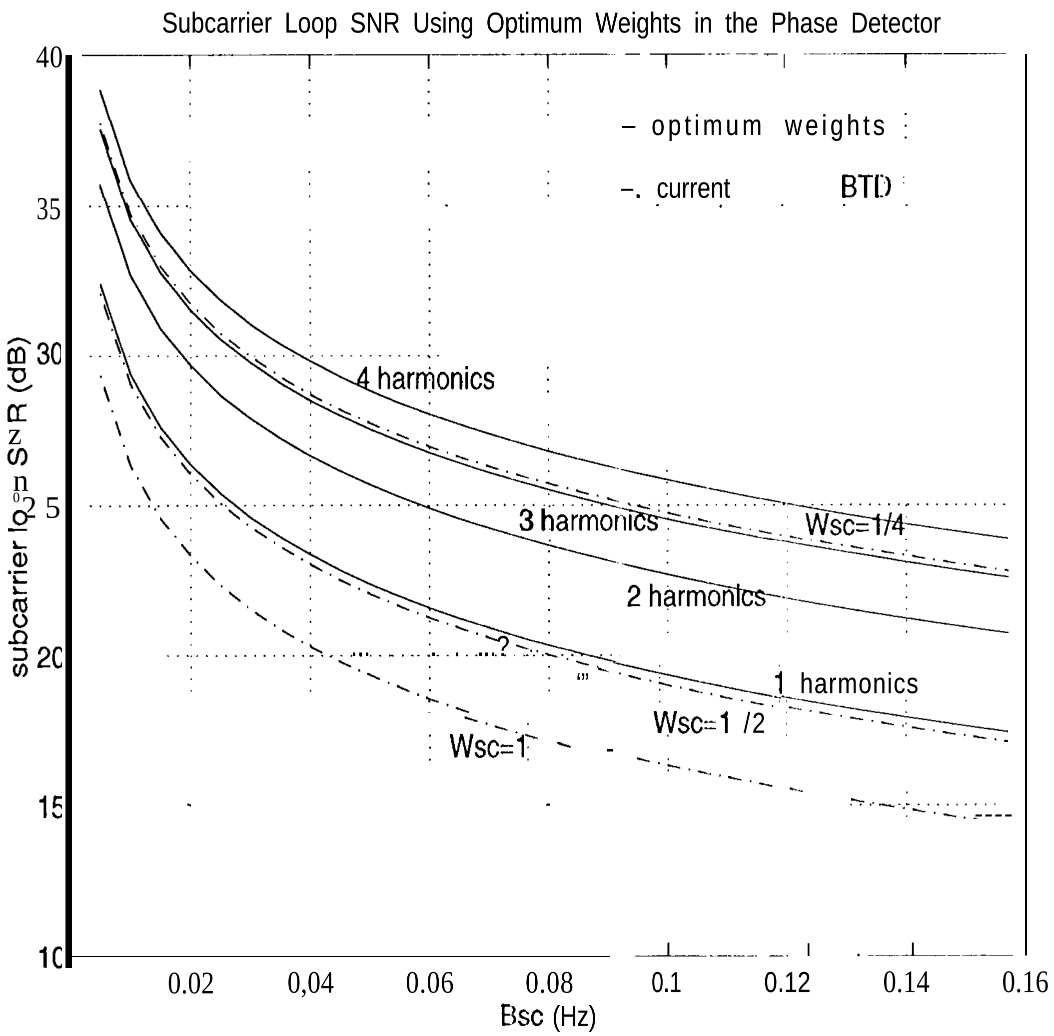

Figure 2: Comparison in loop SNR using the optimum weights in the phase detector and using the initial BTD.

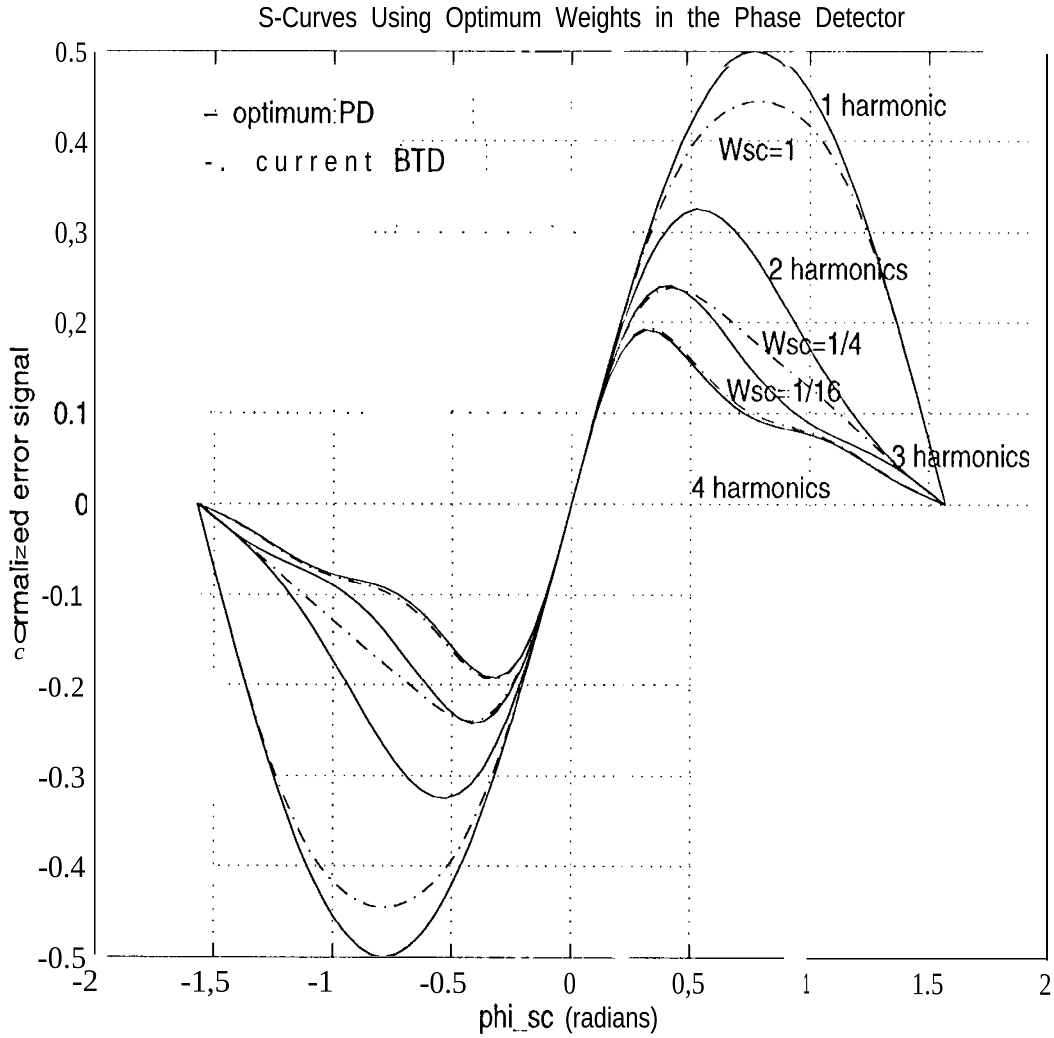

Figure 3: Normalized S-curves.

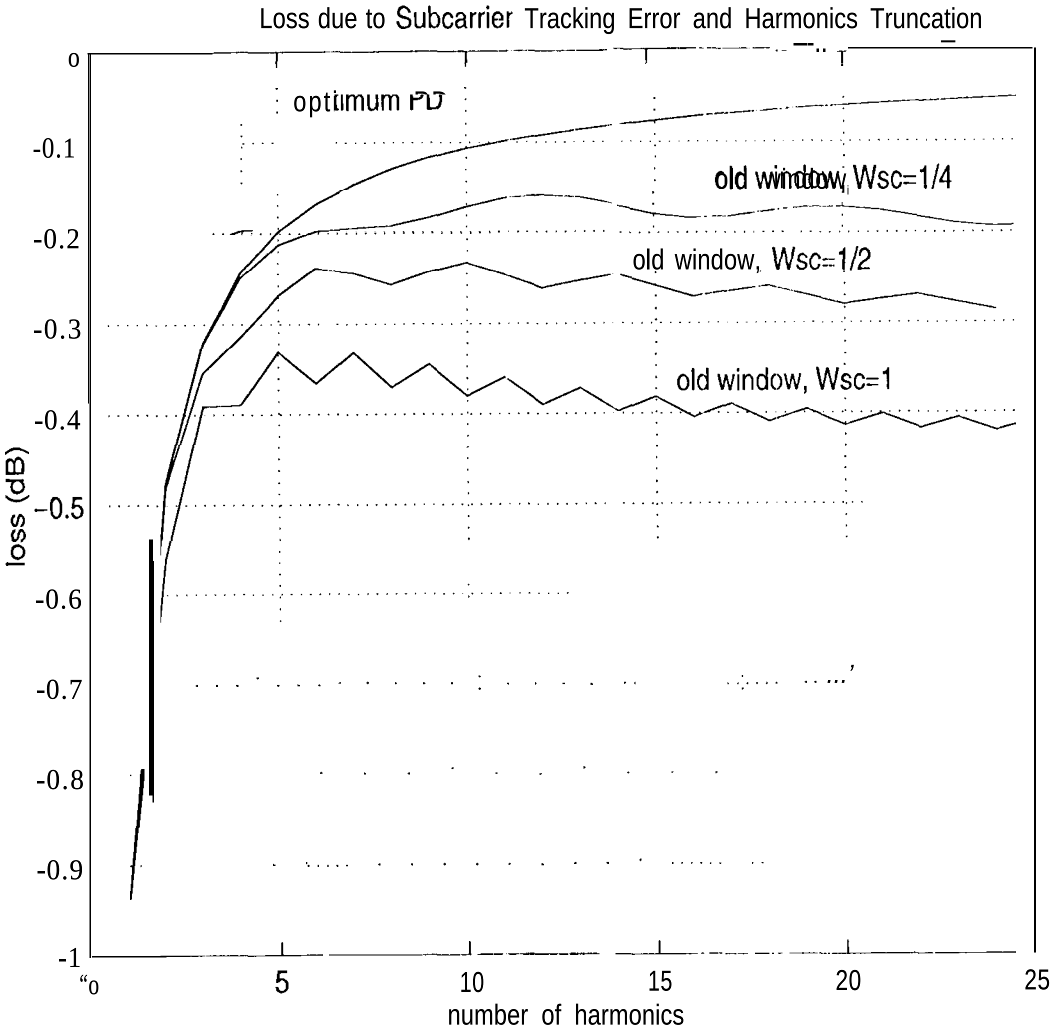

Figure 4: Degradations as function of number of harmonics using the optimum weights.

the first harmonic has a phase jitter with a variance of σ^2, then the n-th harmonic would have a variance of $(n\sigma)^2$. The weight $1/n$ is no longer optimum.

To derive the optimum combining weights, we assume that the harmonics are combined using unknown weights b_n. We then express the SNR in terms of the weights. Differentiating the SNR with respect to the weights and setting the result to zero, we should obtain the optimum weights.

The optimum weights to combine the demodulated harmonics are derived in the appendix to be

$$b_n = \frac{\overline{\cos[(2n+1)\phi_{sc}]}}{\overline{\cos\phi_{sc}}} \cdot \frac{1}{2n+1} \tag{3}$$

When ϕ_{sc} is assumed to have a Tikhonov distribution, we have

$$\overline{\cos(2n+1)\phi_{sc}} = \int_0^{\pi} \frac{\exp(\frac{1}{4}\rho_{sc}\cos\phi_{sc})}{\pi I_0(\rho_{sc}/4)} \cos\left[\frac{2n+1}{2}\phi_{sc}\right] d\phi_{sc}$$

Assuming that we have 4, 6, and 8 harmonics, the degradations in symbol SNR due to the subcarrier loop versus the subcarrier-loop SNR using the optimum combining weights and using the usual $1/n$ weights are compared in Figs. 5 to 7.

4 Approximate Optimum Combining Weights in Demodulation

Since the cosine function is "smooth" in the vicinity of zero, for small phase jitters, $n\phi_{sc}$, the expected value of $\cos(n\phi_{sc})$ can be approximated by

$$\mathcal{E}\{\cos(n\phi_{sc})\} \approx 1 - n^2\frac{\sigma^2}{2} \tag{4}$$

The approximate optimum weights are

$$b_n \approx \frac{1-(2n+1)^2\sigma^2/2}{1-\sigma^2/2} \cdot \frac{1}{2n+1} \tag{5}$$

Note that this approximation is valid only when $n\phi_{sc}$ is small. Using the approximated optimum weights for four harmonics, the symbol SNR degradation is only slightly more than that using the optimum weights as shown in Fig. 5.

5 Conclusion

We presented an optimum way for tracking and demodulating a finite-harmonic subcarrier. We found an optimum phase detector in the sense that the loop SNR

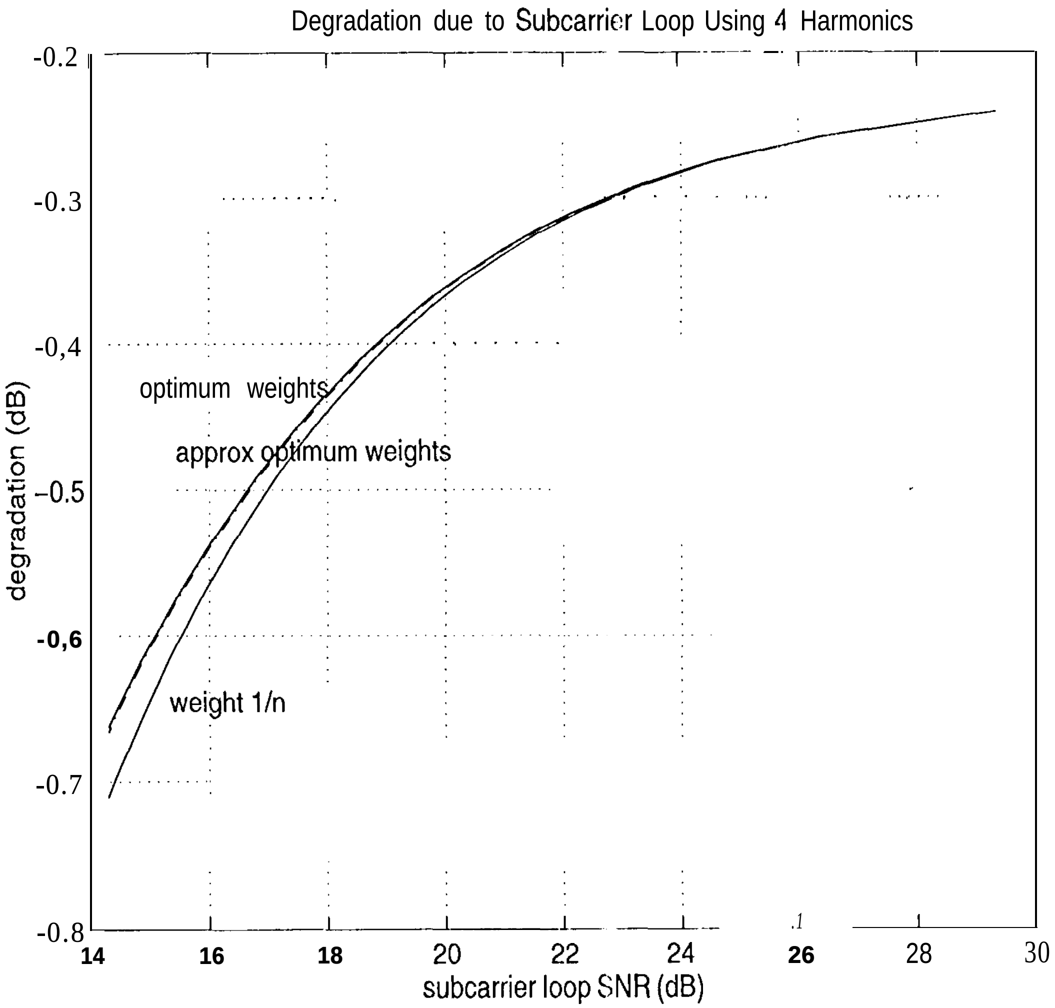

Figure 5: Symbol SNR degradation when using optimum weights.

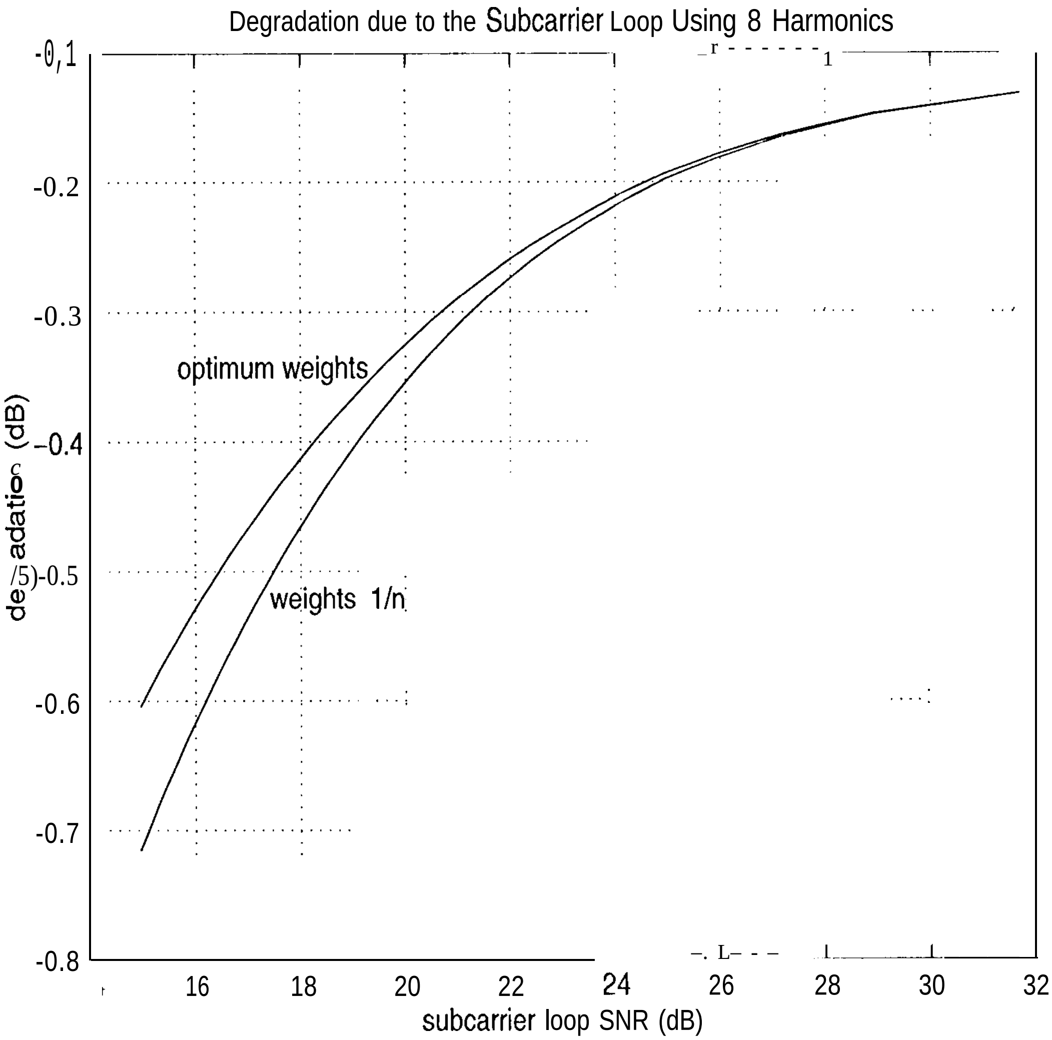

Figure 6: Symbol SNR degradation when using optimum weights.

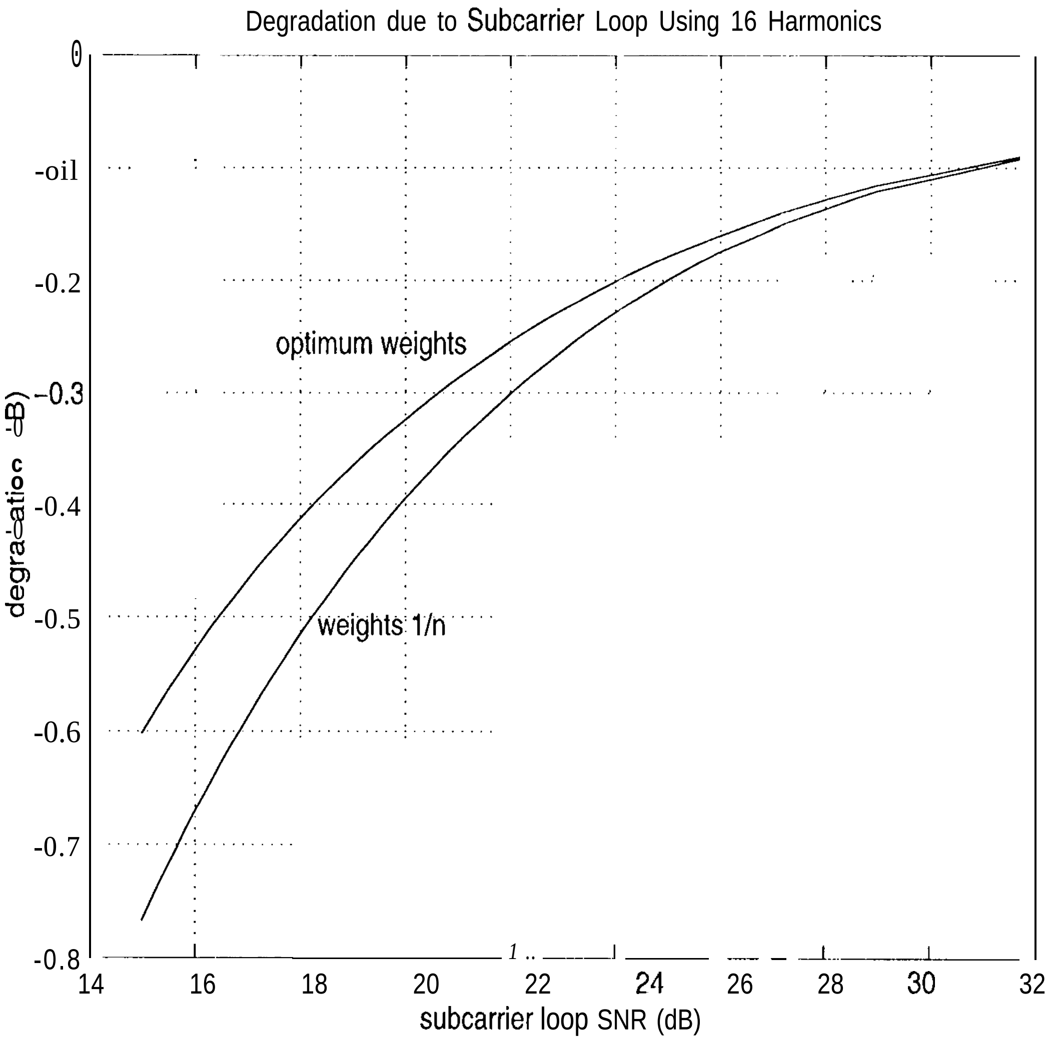

Figure 7: Symbol SNR degradation when using optimum weights.

is maximized. The more harmonics used, the higher loop SNR we obtain. However, the linear region of the phase error signal shrinks with the increase of the number of harmonics. Therefore, this optimum phase detector is only appropriate for a finite number of harmonics. Using the optimum phase detector, the loop SNR is about 9.5 dB higher than that of the initial BTD using window size one, and is about 1 dB higher than that of the initial BTD with window size 1 /4.

For demodulation, we found that the optimum combining weights that account for the losses due to the phase jitter. Compared to using the usual 1/n combining weights, the use of the optimum combining weights can improve the symbol SNR by 0.1 to 0.15 dB at a low loop SNR (15 dB) and high number of harmonics (8 to 16).

References

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Appendix

Derivation of the Optimum Combining Weights in Demodulation

After each of the harmonics of the subcarrier is demodulated, the signals from each harmonic demodulation need to be combined. Assume that the combining weight for the $(2n + 1)$-th harmonic to be b_n, the signal amplitude at the l-th symbol is

$$s = \sqrt{P_d} d_l \cos\phi_c \frac{2}{\pi} \sum_{n=0}^{L-1} b_n \frac{1}{2n+1} \cos[(2n+1)\phi_{sc}] \tag{6}$$

where P_d is the data power, and ϕ_c and ϕ_{sc} are the phase offsets of carrier and subcarrier respectively. The noise variance is

$$\sigma^2 = \sum_{n=0}^{L-1} b_n^2 \frac{N_o}{2} R_{sym} \tag{7}$$

Taking the ratio of the average signal power and the noise variance, we have the average symbol SNR of the combined signal

$$\begin{aligned} SNR &= \frac{\mathcal{E}\{s^2\}}{2\sigma^2} \\ &= \frac{\mathcal{E}\{\frac{4}{\pi^2} P_d \overline{\{\cos^2\phi_c\}} (\sum_{n=0}^{L-1} b_n \frac{\cos[(2n+1)\phi_{sc}]}{2n+1})^2\}}{\sum_{n=0}^{L-1} b_n^2 N_o R_{sym}} \end{aligned} \tag{8}$$

Differentiating the symbol SNR with respect to $b_k, k = 0, \ldots, L-1$, we have

$$\begin{aligned} \frac{\partial(SNR)}{\partial b_k} &= \frac{P_d \overline{\cos^2 \phi_c} \frac{4}{\pi^2}}{(\sum_{n=0}^{L-1} b_n^2 N_o R_{sym})^2} \\ &\quad \mathcal{E}\left\{ \left[2 \sum_{n=0}^{L-1} b_n \frac{\cos[(2n+1)\phi_{sc}]}{2n+1} \right] \frac{\cos[(2k+1)\phi_{sc}]}{2k+1} \sum_{n=0}^{L-1} b_n^2 N_o R_{sym} \right. \\ &\quad \left. - \left[\sum_{n=0}^{L-1} b_n \frac{\cos[(2n+1)\phi_{sc}]}{2n+1} \right]^2 2 b_k N_o R_{sym} \right\} \\ &= 0 \end{aligned} \tag{9}$$

Simplifying the above equation, we have

$$\mathcal{E}\left\{ \frac{\cos[(2k+1)\phi_{sc}]}{2k+1} \sum_{n=0}^{L-1} b_n^2 - \sum_{n=0}^{L-1} b_n \frac{\cos[(2n+1)\phi_{sc}]}{2n+1} b_k \right\} = 0 \tag{10}$$

Letting $k = 0$, and $b_0 = 1$, we have

$$\overline{\cos\phi_{sc}} \sum_{n=0}^{L-1} b_n^2 - \sum_{n=0}^{L-1} b_n \frac{\overline{\cos[(2n+1)\phi_{sc}]}}{2n+1} = 0 \tag{11}$$

That is,

$$\sum_{n=0}^{L-1} b_n \left[\overline{\cos\phi_{sc}} b_n - \frac{\overline{\cos[(2n+1)\phi_{sc}]}}{2n+1} \right] = 0 \tag{12}$$

Finally, solving for b_n, we have the optimum combining weights,

$$b_n = \frac{\overline{\cos[(2n+1)\phi_{sc}]}}{\overline{\cos\phi_{sc}}} \frac{1}{2n+1} \tag{13}$$